Qualifying exam syllabus

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July 31, 2016

Committee

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MAJOR TOPIC: PROBABILITY THEORY (PROBABILITY)

- **Preliminaries** σ -algebras, Dynkin's π - λ theorem, Independence, Borel–Cantelli lemma, Kolmogorov's 0-1 law, Kolmogorov's maximal inequality, Strong and weak laws of large numbers
- **Central limit theorems** Weak convergence, Uniform integrability, Characteristic functions, I.I.D. central limit theorem, Lindeberg–Feller CLT
- **Conditioning** Conditional probability, Conditional expectation, Conditional independence, Regular conditional probabilities
- **Martinagles** Stopping times, Upcrossing inequality, A.S. convergence, Doob's decomposition, Doob's inequality, L^p convergence, L^1 convergence, Reverse martingale convergence, Optional stopping theorem, Wald's identity
- Markov chains Countable state space, Stationary measures, Convergence theorems, Recurrence and transience, Asymptotic behavior
- References Durrett, Probability: Theory and Examples, Chapters 1-3, 5-6

MAJOR TOPIC: THEORETICAL STATISTICS (PROBABILITY)

- **Exponential family** Densities, Parameters, Moments, Cumulants, Generating functions
- **Estimators and statistics** Risks, Estimators, Sufficient statistic, Complete statistic, Ancillary statistic, Factorization theorem, Minimal sufficient statistic, Basu's theorem, Rao–Blackwell theorem
- Unbiased estimation UMVU estimators, Variance bounds, Fisher information, Cramér–Rao bound, Higher dimension variance bounds

- Bayesian estimation Prior and posterior distribution, conjugate distribution
- Large sample theory Convergence in probability, Convergence in distribution, Central limit theorem, Delta method, Asymptotic relative efficiency
- **Estimating equations** Weak law for random functions, Kullback–Leibler divergence, Consistency of maximum likelihood estimator, limiting distribution for MLE, Confidence interval, asymptotic confidence interval
- Empirical Bayes Empirical Bayes estimator, James-Stein estimator
- **Hypothesis testing** Test function, Power, Significance, Neyman–Pearson lemma, Uniformly most powerful tests, Monotone likelihood ratio, Duality between testing and interval estimation, Generalized Neyman–Pearson lemma, Two-sided hypothesis, Unbiased test
- References Keener, Theoretical Statistics, Chapters 2–4, 6–9, 11

MINOR TOPIC: NUMERICAL LINEAR ALGEBRA (APPLIED MATHEMATICS)

- Linear equation solving Gaussian elimination, Perturbation theory, Blocking algorithms, Special linear systems (real symmetric position definite matrices, symmetric indefinite matrices, general sparse matrices)
- Linear least squares Normal equations, QR decomposition, Singular value decomposition, Householder transformations, Givens rotations, Rank-deficient least square problems, Perturbation theory
- **Nonsymmetric eigenproblem** Power method, Inverse iteration, Orthogonal iteration, QR iteration, Tridiagonal and bidiagonal reduction, Perturbation theory
- **Symmetric eigenproblem** Tridiagonal QR iteration, Rayleigh quotient iteration, Divide-and-conquer, Bisection and inverse iteration, Perturbation theory
- **References** Demmel, Applied Linear Algebra, Chapters 2–5